

# Pari-GP reference card

(PARI-GP version 2.9.0)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

## Help

describe function	?function
extended description	??keyword
list of relevant help topics	???pattern
name of GP-1.39 function $f$ in GP-2.*	whatnow( $f$ )

## Input/Output

previous result, the result before	%, %', %'', etc.
$n$ -th result since startup	% $n$
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{seq <sub>1</sub> ; seq <sub>2</sub> ;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

## Metacommands & Defaults

set default $d$ to $val$	default({ $d$ },{ $val$ })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to $n$	\g $n$
set memory debug level to $n$	\gm $n$
set $n$ significant digits / bits	\p $n$ , \pb $n$
set $n$ terms in series	\ps $n$
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r filename

## Debugger / break loop

get out of break loop	break or <C-D>
go up/down $n$ frames	dbg_up({ $n$ }), dbg_down
set break point	breakpoint()
examine object $o$	dbg_x( $o$ )
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

## PARI Types & Input Formats

<b>t_INT</b> . Integers; hex, binary	$\pm 31$ ; $\pm 0x1F$ , $\pm 0b101$
<b>t_REAL</b> . Reals	$\pm 3.14$ , 6.022 E23
<b>t_INTMOD</b> . Integers modulo $m$	Mod( $n$ , $m$ )
<b>t_FRAC</b> . Rational Numbers	$n/m$
<b>t_FFELT</b> . Elt in finite field $F_q$	ffgen( $q$ )
<b>t_COMPLEX</b> . Complex Numbers	$x + y * I$
<b>t_PADIC</b> . $p$ -adic Numbers	$x + O(p^k)$
<b>t_QUAD</b> . Quadratic Numbers	$x + y * \text{quadgen}(D)$
<b>t_POLMOD</b> . Polynomials modulo $g$	Mod( $f$ , $g$ )
<b>t_POL</b> . Polynomials	$a * x^n + \dots + b$
<b>t_SER</b> . Power Series	$f + O(x^k)$
<b>t_RFRAC</b> . Rational Functions	$f/g$
<b>t_QFI</b> / <b>t_QFR</b> . Imag/Real binary quad. form	Qfb( $a, b, c, \{d\}$ )
<b>t_VEC</b> / <b>t_COL</b> . Row/Column Vectors	[ $x, y, z$ ], [ $x, y, z$ ]~
<b>t_VEC</b> integer range	[1..10]

<b>t_VECSMALL</b> . Vector of small ints	Vecsmall([ $x, y, z$ ])
<b>t_MAT</b> . Matrices	[ $a, b; c, d$ ]
<b>t_LIST</b> . Lists	List([ $x, y, z$ ])
<b>t_STR</b> . Strings	"abc"
<b>t_INFINITY</b> . $\pm\infty$	+oo, -oo

## Reserved Variable Names

$\pi = 3.14\dots$ , $\gamma = 0.57\dots$ , $C = 0.91\dots$	Pi, Euler, Catalan
square root of $-1$	I
Landau's big-oh notation	O

## Information about an Object

PARI type of object $x$	type( $x$ )
length of $x$ / size of $x$ in memory	# $x$ , sizebyte( $x$ )
real precision / bit precision of $x$	precision( $x$ ), bitprecision
$p$ -adic, series prec. of $x$	padicprec( $x$ ), serprec

## Operators

basic operations	+, -, *, /, ^, sqr
$i=i+1$ , $i=i-1$ , $i=i*j$ , ...	i++, i--, i*=j,...
euclidean quotient, remainder	$x \backslash y$ , $x \backslash y$ , $x \% y$ , divrem( $x, y$ )
shift $x$ left or right $n$ bits	$x << n$ , $x >> n$ or shift( $x, \pm n$ )
multiply by $2^n$	shiftmul( $x, n$ )
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitnegimply
sign of $x = -1, 0, 1$	sign( $x$ )
maximum/minimum of $x$ and $y$	max, min( $x, y$ )
derivative of $f$	$f'$
differential operator	diffop( $f, v, d, \{n = 1\}$ )
quote operator (formal variable)	' $x$
assignment	$x = value$
simultaneous assignment $x \leftarrow v_1, y \leftarrow v_2$	[ $x, y$ ] = $v$

## Select Components

$n$ -th component of $x$	component( $x, n$ )
$n$ -th component of vector/list $x$	$x[n]$
components $a, a + 1, \dots, b$ of vector $x$	$x[a..b]$
$(m, n)$ -th component of matrix $x$	$x[m, n]$
row $m$ or column $n$ of matrix $x$	$x[m, ]$ , $x[, n]$
numerator/denominator of $x$	numerator( $x$ ), denominator

## Random Numbers

random integer/prime in $[0, N[$	random( $N$ ), randomprime
get/set random seed	getrand, setrand( $s$ )

## Conversions

to vector, matrix, vec. of small ints	Col/Vec, Mat, Vecsmall
to list, set, map, string	List, Set, Map, Str
create PARI object ( $x \bmod y$ )	Mod( $x, y$ )
make $x$ a polynomial of $v$	Pol( $x, \{v\}$ )
as Pol, etc., starting with constant term	Polrev, Vecrev, Colrev
make $x$ a power series of $v$	Ser( $x, \{v\}$ )
string from bytes / from format+args	Strchr, Strprintf
TeX string	Strtex( $x$ )
convert $x$ to simplest possible type	simplify( $x$ )
object $x$ with real precision $n$	precision( $x, n$ )
object $x$ with bit precision $n$	bitprecision( $x, n$ )
set precision to $p$ digits in dynamic scope	localprec( $p$ )
set precision to $p$ bits in dynamic scope	localbitprec( $p$ )

## Conjugates and Lifts

conjugate of a number $x$	conj( $x$ )
norm of $x$ , product with conjugate	norm( $x$ )
$L^p$ norm of $x$ ( $L^\infty$ if no $p$ )	normlp( $x, \{p\}$ )
square of $L^2$ norm of $x$	norml2( $x$ )
lift of $x$ from Mods and $p$ -adics	lift, centerlift( $x$ )
recursive lift	liftall
lift all <b>t_INT</b> and <b>t_PADIC</b> ( $\rightarrow$ <b>t_INT</b> )	liftint
lift all <b>t_POLMOD</b> ( $\rightarrow$ <b>t_POL</b> )	liftpol

## Lists, Sets & Maps

**Sets** (= row vector with strictly increasing entries w.r.t. **cmp**)

intersection of sets $x$ and $y$	setintersect( $x, y$ )
set of elements in $x$ not belonging to $y$	setminus( $x, y$ )
union of sets $x$ and $y$	setunion( $x, y$ )
does $y$ belong to the set $x$	setsearch( $x, y, \{flag\}$ )
set of all $f(x, y)$ , $x \in X$ , $y \in Y$	setbinop( $f, X, Y$ )
is $x$ a set ?	setisset( $x$ )

**Lists**. create empty list:  $L = \text{List}()$

append $x$ to list $L$	listput( $L, x, \{i\}$ )
remove $i$ -th component from list $L$	listpop( $L, \{i\}$ )
insert $x$ in list $L$ at position $i$	listinsert( $L, x, i$ )
sort the list $L$ in place	listsort( $L, \{flag\}$ )

**Maps**. create empty dictionary:  $M = \text{Map}()$

attach value $v$ to key $k$	mapput( $M, k, v$ )
recover value attach to key $k$ or error	mapget( $M, k$ )
is key $k$ in the dict ? (set $v$ to $M(k)$ )	mapisdefined( $M, k, \{\&v\}$ )
remove $k$ from map domain	mapdelete( $M, k$ )

## GP Programming

### User functions and closures

$x, y$  are formal parameters;  $y$  defaults to **Pi** if parameter opitted;  $z, t$  are local variables (lexical scope),  $z$  initialized to 1.

```
fun(x, y=Pi) = my(z=1, t); seq
fun = (x, y=Pi) -> my(z=1, t); seq
attach a help message to f      addhelp(f)
undefine symbol s (also kills help)  kill(s)
Control Statements ( $X$ : formal parameter in expression  $seq$ )
if  $a \neq 0$ , evaluate  $seq_1$ , else  $seq_2$       if(a, {seq1}, {seq2})
eval.  $seq$  for  $a \leq X \leq b$                   for(X = a, b, seq)
...for primes  $a \leq X \leq b$                   forprime(X = a, b, seq)
...for composites  $a \leq X \leq b$               forcomposite(X = a, b, seq)
...for  $a \leq X \leq b$  stepping  $s$               forstep(X = a, b, s, seq)
...for  $X$  dividing  $n$                           fordiv(n, X, seq)
multivariable for, lex ordering             forvec(X = v, seq)
loop over partitions of  $n$                    forpart(p = n, seq)
loop over vectors  $v$ ,  $q(v) \leq B$ ;  $q > 0$        forqfvec(v, q, b, seq)
loop over  $H < G$  finite abelian group        forsubgroup(H = G)
```

evaluate $seq$ until $a \neq 0$	until( $a, seq$ )
while $a \neq 0$ , evaluate $seq$	while( $a, seq$ )
exit $n$ innermost enclosing loops	break({ $n$ })
start new iteration of $n$ -th enclosing loop	next({ $n$ })
return $x$ from current subroutine	return({ $x$ })

### Exceptions, warnings

raise an exception / warn	error(), warning()
type of error message $E$	errname( $E$ )
try $seq_1$ , evaluate $seq_2$ on error	iferr( $seq_1, E, seq_2$ )

Functions with closure arguments / results

select from  $v$  according to  $f$  `select( $f, v$ )`  
apply  $f$  to all entries in  $v$  `apply( $f, v$ )`  
evaluate  $f(a_1, \dots, a_n)$  `call( $f, a$ )`  
evaluate  $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$  `fold( $f, a$ )`  
calling function as closure `self()`

Sums & Products

sum  $X = a$  to  $X = b$ , initialized at  $x$  `sum( $X = a, b, expr, \{x\}$ )`  
sum entries of vector  $v$  `vecsum( $v$ )`  
sum  $expr$  over divisors of  $n$  `sumdiv( $n, X, expr$ )`  
... assuming  $expr$  multiplicative `sumdivmult( $n, X, expr$ )`  
product  $a \leq X \leq b$ , initialized at  $x$  `prod( $X = a, b, expr, \{x\}$ )`  
product over primes  $a \leq X \leq b$  `prodeuler( $X = a, b, expr$ )`

Sorting

sort  $x$  by  $k$ -th component `vecsort( $x, \{k\}, \{fl = 0\}$ )`  
min.  $m$  of  $x$  ( $m = x[i]$ ), max. `vecmin( $x, \{\&i\}$ ), vecmax`  
does  $y$  belong to  $x$ , sorted wrt.  $f$  `vecsearch( $x, y, \{f\}$ )`

Input/Output

print with/without  $\backslash n$ , T<sub>E</sub>X format `print, print1, printtex`  
print fields with separator `printsep( $sep, \dots$ ), printsep1`  
formatted printing `printf()`  
write  $args$  to file `write, write1, writetex( $file, args$ )`  
write  $x$  in binary format `writebin( $file, x$ )`  
read file into GP `read( $\{file\}$ )`  
... return as vector of lines `readvec( $\{file\}$ )`  
... return as vector of strings `readstr( $\{file\}$ )`  
read a string from keyboard `input()`

Timers

CPU time in  $ms$  and reset timer `gettime()`  
CPU time in  $ms$  since gp startup `getabstime()`  
time in  $ms$  since UNIX Epoch `getwalltime()`  
timeout command after  $s$  seconds `alarm( $s, expr$ )`

Interface with system

allocates a new stack of  $s$  bytes `allocatemem( $\{s\}$ )`  
alias  $old$  to  $new$  `alias( $new, old$ )`  
install function from library `install( $f, code, \{gpf\}, \{lib\}$ )`  
execute system command  $a$  `system( $a$ )`  
as above, feed result to GP `extern( $a$ )`  
as above, return GP string `externstr( $a$ )`  
get \$VAR from environment `getenv("VAR")`  
expand env. variable in string `Strexpand( $x$ )`

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables and must be free of side effects. Enabled if threading engine is not *single* in gp header.  
evaluate  $f$  on  $x[1], \dots, x[n]$  `parapply( $f, x$ )`  
evaluate closures  $f[1], \dots, f[n]$  `pareval( $f$ )`  
as **select** `parselect( $f, A, \{flag\}$ )`  
as **sum** `parsum( $i = a, b, expr, \{x\}$ )`  
as **vector** `parvector( $n, i, \{expr\}$ )`  
eval  $f$  for  $i = a, \dots, b$  `parfor( $i = a, \{b\}, f, \{r\}, \{f_2\}$ )`  
... for  $p$  prime in  $[a, b]$  `parforprime( $p = a, \{b\}, f, \{r\}, \{f_2\}$ )`  
... multivariate `parforvec( $X = v, f, \{r\}, \{f_2\}, \{flag\}$ )`  
declare  $x$  as inline (allows to use as global) `inline( $x$ )`  
stop inlining `uninline()`

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Linear Algebra

dimensions of matrix  $x$  `matsize( $x$ )`  
concatenation of  $x$  and  $y$  `concat( $x, \{y\}$ )`  
extract components of  $x$  `vecextract( $x, y, \{z\}$ )`  
transpose of vector or matrix  $x$  `mattranspose( $x$ )` or  $x \sim$   
adjoint of the matrix  $x$  `matadjoint( $x$ )`  
eigenvectors/values of matrix  $x$  `mateigen( $x$ )`  
characteristic/minimal polynomial of  $x$  `charpoly( $x$ ), minpoly`  
trace/determinant of matrix  $x$  `trace( $x$ ), matdet`  
Frobenius form of  $x$  `matfrobenius( $x$ )`  
QR decomposition `matqr( $x$ )`  
apply `matqr`'s transform to  $v$  `mathouseholder( $Q, v$ )`

Constructors & Special Matrices

$\{g(x): x \in v \text{ s.t. } f(x)\}$  `[ $g(x) \mid x \leftarrow v, f(x)$ ]`  
 $\{x: x \in v \text{ s.t. } f(x)\}$  `[ $x \mid x \leftarrow v, f(x)$ ]`  
 $\{g(x): x \in v\}$  `[ $g(x) \mid x \leftarrow v$ ]`  
row vec. of  $expr$  eval'ed at  $1 \leq i \leq n$  `vector( $n, \{i\}, \{expr\}$ )`  
col. vec. of  $expr$  eval'ed at  $1 \leq i \leq n$  `vectorv( $n, \{i\}, \{expr\}$ )`  
vector of small ints `vectorsmall( $n, \{i\}, \{expr\}$ )`  
 $[c, c \cdot x, \dots, c \cdot x^n]$  `powers( $x, n, \{c = 1\}$ )`  
matrix  $1 \leq i \leq m, 1 \leq j \leq n$  `matrix( $m, n, \{i\}, \{j\}, \{expr\}$ )`  
define matrix by blocks `matconcat( $B$ )`  
diagonal matrix with diagonal  $x$  `matdiagonal( $x$ )`  
is  $x$  diagonal? `matisdiagonal( $x$ )`  
 $x \cdot \text{matdiagonal}(d)$  `matmuldiagonal( $x, d$ )`  
 $n \times n$  identity matrix `matid( $n$ )`  
Hessenberg form of square matrix  $x$  `mathess( $x$ )`  
 $n \times n$  Hilbert matrix  $H_{ij} = (i + j - 1)^{-1}$  `mathilbert( $n$ )`  
 $n \times n$  Pascal triangle `matpascal( $n - 1$ )`  
companion matrix to polynomial  $x$  `matcompanion( $x$ )`  
Sylvester matrix of  $x$  `polsylvestermatrix( $x$ )`

Gaussian elimination

kernel of matrix  $x$  `matker( $x, \{flag\}$ )`  
intersection of column spaces of  $x$  and  $y$  `matintersect( $x, y$ )`  
solve  $M * X = B$  ( $M$  invertible) `matsolve( $M, B$ )`  
as solve, modulo  $D$  (col. vector) `matsolvemod( $M, D, B$ )`  
one sol of  $M * X = B$  `matinverseimage( $M, B$ )`  
basis for image of matrix  $x$  `matimage( $x$ )`  
columns of  $x$  *not* in `matimage` `matimagecompl( $x$ )`  
supplement columns of  $x$  to get basis `mat supplement( $x$ )`  
rows, cols to extract invertible matrix `matindexrank( $x$ )`  
rank of the matrix  $x$  `matrank( $x$ )`

Lattices & Quadratic Forms

Quadratic forms

evaluate  ${}^t x Q y$  `qfeval( $\{Q = id\}, x, y$ )`  
evaluate  ${}^t x Q x$  `qfeval( $\{Q = id\}, x$ )`  
signature of quad form  ${}^t y * x * y$  `qfsign( $x$ )`  
decomp into squares of  ${}^t y * x * y$  `qfgaussred( $x$ )`  
eigenvalues/vectors for real symmetric  $x$  `qfjacobi( $x$ )`

HNF and SNF

upper triangular Hermite Normal Form `mathnf( $x$ )`  
HNF of  $x$  where  $d$  is a multiple of  $\det(x)$  `mathnfmod( $x, d$ )`  
multiple of  $\det(x)$  `matdetint( $x$ )`  
HNF of  $(x \mid \text{diagonal}(D))$  `mathnfmod( $x, D$ )`  
elementary divisors of  $x$  `mathsnf( $x$ )`  
elementary divisors of  $\mathbf{Z}[a]/(f'(a))$  `poldiscreduced( $f$ )`  
integer kernel of  $x$  `matkerint( $x$ )`  
 $\mathbf{Z}$ -module  $\leftrightarrow$   $\mathbf{Q}$ -vector space `matrixqz( $x, p$ )`

Lattices

LLL-algorithm applied to columns of  $x$  `qflll( $x, \{flag\}$ )`  
... for Gram matrix of lattice `qflllgram( $x, \{flag\}$ )`  
find up to  $m$  sols of `qfnorm( $x, y\} \leq b$  qfminim( $x, b, m$ )  
 $v, v[i] :=$  number of  $y$  s.t. qfnorm( $x, y\} = i$  qfrep( $x, B, \{flag\}$ )  
perfection rank of  $x$  qfperfection( $x$ )  
find isomorphism between  $q$  and  $Q$  qfism( $q, Q$ )  
precompute for isomorphism test with  $q$  qfisminit( $q$ )  
automorphism group of  $q$  qfauto( $q$ )  
convert qfauto for GAP/Magma qfautoexport( $G, \{flag\}$ )  
orbits of  $V$  under  $G \subset \text{GL}(V)$  qforbits( $G, V$ )`

Polynomials & Rational Functions

all defined polynomial variables `variables()`  
get var. of highest priority (higher than  $v$ ) `varhigher( $name, \{v\}$ )`  
... of lowest priority (lower than  $v$ ) `varlower( $name, \{v\}$ )`

Coefficients, variables and basic operators

degree of  $f$  `poldegree( $f$ )`  
coeff. of degree  $n$  of  $f$ , leading coeff. `polcoeff( $f, n$ ), pollead`  
main variable / all variables in  $f$  `variable( $f$ ), variables( $f$ )`  
replace  $x$  by  $y$  in  $f$  `subst( $f, x, y$ )`  
evaluate  $f$  replacing vars by their value `eval( $f$ )`  
replace polynomial expr.  $T(x)$  by  $y$  in  $f$  `substpol( $f, T, y$ )`  
replace  $x_1, \dots, x_n$  by  $y_1, \dots, y_n$  in  $f$  `substvec( $f, x, y$ )`  
reciprocal polynomial  $x^{\deg f} f(1/x)$  `polrecip( $f$ )`  
gcd of coefficients of  $f$  `content( $f$ )`  
derivative of  $f$  w.r.t.  $x$  `deriv( $f, \{x\}$ )`  
formal integral of  $f$  w.r.t.  $x$  `intformal( $f, \{x\}$ )`  
formal sum of  $f$  w.r.t.  $x$  `sumformal( $f, \{x\}$ )`

Constructors & Special Polynomials

interpolating pol. eval. at  $a$  `polinterpolate( $X, \{Y\}, \{a\}$ )`  
 $P_n, T_n/U_n, H_n$  `pollegendre, polchebyshev, polhermite`  
 $n$ -th cyclotomic polynomial  $\Phi_n$  `polcyclo( $n, \{v\}$ )`  
return  $n$  if  $f = \Phi_n$ , else 0 `poliscyclo( $f$ )`  
is  $f$  a product of cyclotomic polynomials? `poliscycloprod( $f$ )`  
Zagier's polynomial of index  $(n, m)$  `polzagier( $n, m$ )`

Resultant, elimination

discriminant of polynomial  $f$  `poldisc( $f$ )`  
resultant  $R = \text{Res}_v(f, g)$  `polresultant( $f, g, \{v\}$ )`  
 $[u, v, R], xu + yv = \text{Res}_v(f, g)$  `polresultanttext( $x, y, \{v\}$ )`  
solve Thue equation  $f(x, y) = a$  `thue( $t, a, \{sol\}$ )`  
initialize  $t$  for Thue equation solver `thueinit( $f$ )`

Based on an earlier version by Joseph H. Silverman

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## Roots and Factorization

complex roots of $f$	<code>polroots(<math>f</math>)</code>
number of real roots of $f$ (in $[a, b]$ )	<code>polsturm(<math>f, \{[a, b]\}</math>)</code>
real roots of $f$ (in $[a, b]$ )	<code>polrootsreal(<math>f, \{[a, b]\}</math>)</code>
symmetric powers of roots of $f$ up to $n$	<code>polsym(<math>f, n</math>)</code>
Graeffe transform of $f$ , $g(x^2) = f(x)f(-x)$	<code>polgraeffe(<math>f</math>)</code>
factor $f$	<code>factor(<math>f</math>)</code>
factor $f$ mod $p$ / roots	<code>factormod(<math>f, p</math>), polrootsmod</code>
... using Cantor-Zassenhaus	<code>factorcantor(<math>f, p</math>)</code>
factor $f$ over $\mathbf{F}_{p^a}$ / roots	<code>factorff(<math>f, p, a</math>), polrootsff</code>
factor $f$ over $\mathbf{Q}_p$ / roots	<code>factorpadic(<math>f, p, r</math>), polrootspadic</code>
cyclotomic factors of $f \in \mathbf{Q}[X]$	<code>polcyclofactors(<math>f</math>)</code>
find irreducible $T \in \mathbf{F}_p[x]$ , $\deg T = n$	<code>ffinit(<math>p, n, \{x\}</math>)</code>
$\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$	<code>ffnbirred(<math>q, n</math>)</code>
$p$ -adic root of $f$ congruent to $a$ mod $p$	<code>padicappr(<math>f, a</math>)</code>
Newton polygon of $f$ for prime $p$	<code>newtonpoly(<math>f, p</math>)</code>
Hensel lift $A/\text{lc}(A) = \prod_i B[i] \bmod p^e$	<code>polhensellift(<math>A, B, p, e</math>)</code>
extensions of $\mathbf{Q}_p$ of degree $N$	<code>padicfields(<math>p, N</math>)</code>

## Formal & p-adic Series

truncate power series or $p$ -adic number	<code>truncate(<math>x</math>)</code>
valuation of $x$ at $p$	<code>valuation(<math>x, p</math>)</code>
<b>Dirichlet and Power Series</b>	
Taylor expansion around 0 of $f$ w.r.t. $x$	<code>taylor(<math>f, x</math>)</code>
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	<code>serconvol(<math>a, b</math>)</code>
$f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$	<code>serlaplace(<math>f</math>)</code>
reverse power series $F$ so $F(f(x)) = x$	<code>serreverse(<math>f</math>)</code>
Dirichlet series multiplication / division	<code>dirmul, dirdiv(<math>x, y</math>)</code>
Dirichlet Euler product ( $b$ terms)	<code>direuler(<math>p = a, b, expr</math>)</code>

## Transcendental and $p$ -adic Functions

real, imaginary part of $x$	<code>real(<math>x</math>), imag(<math>x</math>)</code>
absolute value, argument of $x$	<code>abs(<math>x</math>), arg(<math>x</math>)</code>
square/nth root of $x$	<code>sqrt(<math>x</math>), sqrtn(<math>x, n, \{&amp;z\}</math>)</code>
trig functions	<code>sin, cos, tan, cotan, sinc</code>
inverse trig functions	<code>asin, acos, atan</code>
hyperbolic functions	<code>sinh, cosh, tanh, cotanh</code>
inverse hyperbolic functions	<code>asinh, acosh, atanh</code>
$\log(x)$ , $e^x$ , $e^x - 1$	<code>log, exp, expm1</code>
Euler $\Gamma$ function, $\log \Gamma$ , $\Gamma'/\Gamma$	<code>gamma, lngamma, psi</code>
half-integer gamma function $\Gamma(n+1/2)$	<code>gammah(<math>n</math>)</code>
Riemann's zeta $\zeta(s) = \sum n^{-s}$	<code>zeta(<math>s</math>)</code>
multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$	<code>zetamult(<math>s</math>)</code>
incomplete $\Gamma$ function ( $y = \Gamma(s)$ )	<code>incgam(<math>s, x, \{y\}</math>)</code>
complementary incomplete $\Gamma$	<code>incgamc(<math>s, x</math>)</code>
exponential integral $\int_x^\infty e^{-t}/t dt$	<code>eint1(<math>x</math>)</code>
error function $2/\sqrt{\pi} \int_x^\infty e^{-t^2} dt$	<code>erfc(<math>x</math>)</code>
dilogarithm of $x$	<code>dilog(<math>x</math>)</code>
$m$ -th polylogarithm of $x$	<code>polylog(<math>m, x, \{flag\}</math>)</code>
$U$ -confluent hypergeometric function	<code>hyperu(<math>a, b, u</math>)</code>
Bessel $J_n(x)$ , $J_{n+1/2}(x)$	<code>besselj(<math>n, x</math>), besseljh(<math>n, x</math>)</code>
Bessel $I_\nu$ , $K_\nu$ , $H_\nu^1$ , $H_\nu^2$ , $N_\nu$	<code>(bessel)i, k, h1, h2, n</code>
Lambert $W$ : $x$ s.t. $xe^x = y$	<code>lambertw(<math>y</math>)</code>
Teichmuller character of $p$ -adic $x$	<code>teichmuller(<math>x</math>)</code>

## Iterations, Sums & Products

### Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential methods: either a scalar ( $a \in \mathbf{C}$ , regular) or $\pm\infty$ (decreasing at least as $x^{-2}$ ) or		
$(x-a)^{-\alpha}$ singularity	$[a, \alpha]$	
exponential decrease $e^{-\alpha x }$	$[\pm\infty, \alpha]$ , $\alpha > 0$	
slow decrease $ x ^\alpha$	$\dots \alpha < -1$	
oscillating as $\cos(kx)$	$\alpha = k\mathbf{I}$ , $k > 0$	
oscillating as $\sin(kx)$	$\alpha = -k\mathbf{I}$ , $k > 0$	
numerical integration	<code>intnum(<math>x = a, b, f, \{T\}</math>)</code>	
weights $T$ for intnum	<code>intnuminit(<math>a, b, \{m\}</math>)</code>	
weights $T$ incl. kernel $K$	<code>intfuncinit(<math>a, b, K, \{m\}</math>)</code>	
integrate $(2i\pi)^{-1}f$ on circle $ z-a  = R$	<code>intcirc(<math>x = a, R, f, \{T\}</math>)</code>	

### Other integration methods

$n$ -point Gauss-Legendre	<code>intnumgauss(<math>x = a, b, f, \{n\}</math>)</code>
weights for $n$ -point Gauss-Legendre	<code>intnumgaussinit(<math>\{n\}</math>)</code>
Romberg integration (low accuracy)	<code>intnumromb(<math>x = a, b, f, \{flag\}</math>)</code>

### Numerical summation

sum of series $f(n)$ , $n \geq a$ (low accuracy)	<code>suminf(<math>n = a, expr</math>)</code>
sum of alternating/positive series	<code>sumalt, sumpos</code>
sum of series using Euler-Maclaurin	<code>sumnum(<math>n = a, f, \{T\}</math>)</code>
weights for sumnum, $a$ as in DE	<code>sumnuminit(<math>\{\infty, a\}</math>)</code>
sum of series by Monien summation	<code>sumnummonien(<math>n = a, f, \{T\}</math>)</code>
weights for sumnummonien	<code>sumnummonieninit(<math>\{\infty, a\}</math>)</code>

### Products

product $a \leq X \leq b$ , initialized at $x$	<code>prod(<math>X = a, b, expr, \{x\}</math>)</code>
product over primes $a \leq X \leq b$	<code>prodeuler(<math>X = a, b, expr</math>)</code>
infinite product $a \leq X \leq \infty$	<code>prodinf(<math>X = a, expr</math>)</code>

### Other numerical methods

real root of $f$ in $[a, b]$ ; bracketed root	<code>solve(<math>X = a, b, f</math>)</code>
... by interval splitting	<code>solvestep(<math>X = a, b, f, \{flag = 0\}</math>)</code>
limit of $f(t)$ , $t \rightarrow \infty$	<code>limitnum(<math>f, \{k\}, \{\alpha\}</math>)</code>
asymptotic expansion of $f$ at $\infty$	<code>asymnum(<math>f, \{k\}, \{\alpha\}</math>)</code>
numerical derivation w.r.t $x$ : $f'(a)$	<code>derivnum(<math>x = a, f</math>)</code>
evaluate continued fraction $F$ at $t$	<code>contfraceval(<math>F, t, \{L\}</math>)</code>
power series to cont. fraction ( $L$ terms)	<code>contfracinit(<math>S, \{L\}</math>)</code>
Padé approximant (deg. denom. $\leq B$ )	<code>bestapprPade(<math>S, \{B\}</math>)</code>

## Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(<math>x</math>)</code>
bit number $n$ of integer $x$	<code>bittest(<math>x, n</math>)</code>
Hamming weight of integer $x$	<code>hammingweight(<math>x</math>)</code>
digits of integer $x$ in base $B$	<code>digits(<math>x, \{B = 10\}</math>)</code>
sum of digits of integer $x$ in base $B$	<code>sumdigits(<math>x, \{B = 10\}</math>)</code>
integer from digits	<code>fromdigits(<math>v, \{B = 10\}</math>)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round $x$ to nearest integer	<code>round(<math>x, \{&amp;e\}</math>)</code>
truncate $x$	<code>truncate(<math>x, \{&amp;e\}</math>)</code>
gcd/LCM of $x$ and $y$	<code>gcd(<math>x, y</math>), lcm(<math>x, y</math>)</code>
gcd of entries of a vector/matrix	<code>content(<math>x</math>)</code>

## Primes and Factorization

extra prime table	<code>addprimes()</code>
add primes in $v$ to prime table	<code>addprimes(<math>v</math>)</code>
remove primes from prime table	<code>removeprimes(<math>v</math>)</code>
Chebyshev $\pi(x)$ , $n$ -th prime $p_n$	<code>primepi(<math>x</math>), prime(<math>n</math>)</code>
vector of first $n$ primes	<code>primes(<math>n</math>)</code>
smallest prime $\geq x$	<code>nextprime(<math>x</math>)</code>
largest prime $\leq x$	<code>precprime(<math>x</math>)</code>
factorization of $x$	<code>factor(<math>x, \{lim\}</math>)</code>
... selecting specific algorithms	<code>factorint(<math>x, \{flag = 0\}</math>)</code>
$n = df^2$ , $d$ squarefree/fundamental	<code>core(<math>n, \{fl\}</math>), coredisc</code>
recover $x$ from its factorization	<code>factorback(<math>f, \{e\}</math>)</code>
$x \in \mathbf{Z}$ , $ x  \leq X$ , $\gcd(N, P(x)) \geq N$	<code>zncoppersmith(<math>P, N, X, \{B\}</math>)</code>

### Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(<math>n</math>), bigomega</code>
divisors of $n$ / number of divisors $\tau(n)$	<code>divisors(<math>n</math>), numdiv</code>
sum of ( $k$ -th powers of) divisors of $n$	<code>sigma(<math>n, \{k\}</math>)</code>
Möbius $\mu$ -function	<code>moebius(<math>x</math>)</code>
Ramanujan's $\tau$ -function	<code>ramanujantau(<math>x</math>)</code>

### Combinatorics

factorial of $x$	<code>x!</code> or <code>factorial(<math>x</math>)</code>
binomial coefficient $\binom{x}{y}$	<code>binomial(<math>x, y</math>)</code>
Bernoulli number $B_n$ as real/rational	<code>bernreal(<math>n</math>), bernfrac</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol(<math>n, \{x\}</math>)</code>
$n$ -th Fibonacci number	<code>fibonacci(<math>n</math>)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling(<math>n, k, \{flag\}</math>)</code>
number of partitions of $n$	<code>numbpart(<math>n</math>)</code>
$k$ -th permutation on $n$ letters	<code>numtoperm(<math>n, k</math>)</code>
convert permutation to $(n, k)$ form	<code>permtonum(<math>v</math>)</code>

### Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$ , $\mathbf{F}_q^*$

Euler $\phi$ -function	<code>eulerphi(<math>x</math>)</code>
multiplicative order of $x$ (divides $\phi$ )	<code>znorder(<math>x, \{o\}</math>), fforder</code>
primitive root mod $q$ / $x \bmod q$	<code>znprimroot(<math>q</math>), fprimroot(<math>x</math>)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(<math>n</math>)</code>
discrete logarithm of $x$ in base $g$	<code>znlog(<math>x, g, \{o\}</math>), fflag</code>
Kronecker-Legendre symbol $\left(\frac{x}{y}\right)$	<code>kronecker(<math>x, y</math>)</code>
quadratic Hilbert symbol (at $p$ )	<code>hilbert(<math>x, y, \{p\}</math>)</code>

### Miscellaneous

integer square / $n$ -th root of $x$	<code>sqrtint(<math>x</math>), sqrtntint(<math>x, n</math>)</code>
largest integer $e$ s.t. $b^e \leq b$ , $e = \lfloor \log_b(b) \rfloor$	<code>logint(<math>x, b, \{&amp;z\}</math>)</code>
CRT: solve $z \equiv x$ and $z \equiv y$	<code>chinese(<math>x, y</math>)</code>
minimal $u, v$ so $xu + yv = \gcd(x, y)$	<code>gcdext(<math>x, y</math>)</code>
continued fraction of $x$	<code>contfrac(<math>x, \{b\}, \{lmax\}</math>)</code>
last convergent of continued fraction $x$	<code>contfracpnqn(<math>x</math>)</code>
rational approximation to $x$ (den. $\leq B$ )	<code>bestappr(<math>x, \{B\}k</math>)</code>

## Characters

Let  $cyc = [d_1, \dots, d_k]$  represent an abelian group  $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$  or any structure  $G$  affording a `.cyc` method; e.g. `idealstar(,g)` for Dirichlet characters. A character  $\chi$  is coded by  $[c_1, \dots, c_k]$  such that  $\chi(g_j) = e(n_j/d_j)$ .

$\chi \cdot \psi$ ; $\chi^{-1}$ ; $\chi \cdot \psi^{-1}$	<code>charmul, charconj, chardiv</code>
order of $\chi$	<code>charorder(<math>cyc, \chi</math>)</code>
kernel of $\chi$	<code>charker(<math>cyc, \chi</math>)</code>
$\chi(x)$ , $G$ a GP group structure	<code>chareval(<math>G, \chi, x, \{z\}</math>)</code>

Dirichlet Characters

initialize  $G = (\mathbf{Z}/q\mathbf{Z})^*$   $G = \text{idealstar}(,q)$   
is  $\chi$  odd?  $\text{zncharisodd}(G,\chi)$   
real  $\chi \rightarrow$  Kronecker symbol  $(D/. )$   $\text{znchartokronecker}(G,\chi)$   
induce  $\chi \in \hat{G}$  to  $\mathbf{Z}/N\mathbf{Z}$   $\text{zncharinduce}(G,chi,N)$

Conrey labelling

Conrey label  $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$  character  $\text{znconreychar}(G,m)$   
character  $\rightarrow$  Conrey label  $\text{znconreyexp}(G,\chi)$   
log on Conrey generators  $\text{znconreylog}(G,m)$   
conductor of  $\chi$  ( $\chi_0$  primitive)  $\text{znconreyconductor}(G,\chi,\{\chi_0\})$

True-False Tests

is  $x$  the disc. of a quadratic field?  $\text{isfundamental}(x)$   
is  $x$  a prime?  $\text{isprime}(x)$   
is  $x$  a strong pseudo-prime?  $\text{ispseudoprime}(x)$   
is  $x$  square-free?  $\text{issquarefree}(x)$   
is  $x$  a square?  $\text{issquare}(x,\{\&n\})$   
is  $x$  a perfect power?  $\text{ispower}(x,\{k\},\{\&n\})$   
is  $x$  a perfect power of a prime? ( $x = p^n$ )  $\text{isprimepower}(x,\&n\})$   
... of a pseudoprime?  $\text{ispseudoprimepower}(x,\&n\})$   
is  $x$  powerful?  $\text{ispowerful}(x)$   
is  $x$  a totient? ( $x = \varphi(n)$ )  $\text{istotient}(x,\{\&n\})$   
is  $x$  a polygonal number? ( $x = P(s,n)$ )  $\text{ispolygonal}(x,s,\{\&n\})$   
is  $pol$  irreducible?  $\text{polisirreducible}(pol)$

Graphic Functions

crude graph of  $expr$  between  $a$  and  $b$   $\text{plot}(X = a,b,expr)$   
**High-resolution plot** (immediate plot)  
plot  $expr$  between  $a$  and  $b$   $\text{ploth}(X = a,b,expr,\{flag\},\{n\})$   
plot points given by lists  $lx, ly$   $\text{plothraw}(lx,ly,\{flag\})$   
terminal dimensions  $\text{plothsizes}()$

Rectwindow functions

init window  $w$ , with size  $x,y$   $\text{plotinit}(w,x,y)$   
erase window  $w$   $\text{plotkill}(w)$   
copy  $w$  to  $w_2$  with offset  $(dx,dy)$   $\text{plotcopy}(w,w_2,dx,dy)$   
clips contents of  $w$   $\text{plotclip}(w)$   
scale coordinates in  $w$   $\text{plotscale}(w,x_1,x_2,y_1,y_2)$   
 $\text{ploth}$  in  $w$   $\text{plotrecth}(w,X = a,b,expr,\{flag\},\{n\})$   
 $\text{plothraw}$  in  $w$   $\text{plotrecthraw}(w,data,\{flag\})$   
draw window  $w_1$  at  $(x_1,y_1), \dots$   $\text{plotdraw}([[w_1,x_1,y_1],\dots])$

Low-level Rectwindow Functions

set current drawing color in  $w$  to  $c$   $\text{plotcolor}(w,c)$   
current position of cursor in  $w$   $\text{plotcursor}(w)$   
write  $s$  at cursor's position  $\text{plotstring}(w,s)$   
move cursor to  $(x,y)$   $\text{plotmove}(w,x,y)$   
move cursor to  $(x+dx,y+dy)$   $\text{plotrmove}(w,dx,dy)$   
draw a box to  $(x_2,y_2)$   $\text{plotbox}(w,x_2,y_2)$   
draw a box to  $(x+dx,y+dy)$   $\text{plotrbox}(w,dx,dy)$   
draw polygon  $\text{plotlines}(w,lx,ly,\{flag\})$   
draw points  $\text{plotpoints}(w,lx,ly)$   
draw line to  $(x+dx,y+dy)$   $\text{plotrline}(w,dx,dy)$   
draw point  $(x+dx,y+dy)$   $\text{plotrpoint}(w,dx,dy)$   
draw point  $(x+dx,y+dy)$   $\text{plotrpoint}(w,dx,dy)$

Postscript Functions

as  $\text{ploth}$   $\text{psploth}(X = a,b,expr,\{flag\},\{n\})$   
as  $\text{plothraw}$   $\text{psplothraw}(lx,ly,\{flag\})$   
as  $\text{plotdraw}$   $\text{psdraw}([[w_1,x_1,y_1],\dots])$

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